Flutter of Clamped Skew Panels in Supersonic Flow

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The supersonic panel flutter problem of clamped skew panels with in-plane forces is formulated on the basis of the classical, small deflection, thin plate theory using oblique coordinates. The two-dimensional, static approximation is made use of for the aerodynamic loading. Galerkin method with the flutter mode represented in terms of a double series of beam characteristic functions is employed. Results of numerical calculations made for unstressed panels for different combinations of side ratio, angle of skew, and angle of yaw are presented here. The majority of the calculations were made using 16 terms in the series. Convergence is examined in a few typical cases. The dynamic pressure parameter for flutter is found to increase monotonically with the angle of skew for side ratio 1 and to decrease initially before beginning to increase for side ratio 0.5. The results are also compared with those obtained earlier for simply supported panels.

Nomenclature

a,b = panel dimensions along the x_1 and y_1 axes, respectively
C_{rs} = coefficient in the series expansion of deflection
D = plate rigidity, $Eh^3/12(1 - \nu^2)$
E = Young's modulus of the material of the panel
E_{mnrs} = element of the matrix $[E_{mnrs}]$
h = plate thickness
$I_{mr}^{(1)},I_{mr}^{(2)},I_{mr}^{(3)},I_{mr}^{(4)}$ = integrals defined in text, Eq. (11)
$\bar{k}^{*2}$ = frequency parameter, $\rho h\omega^2 a^4 \cos^4\psi/D\pi^4$
$l(x,y,t)$ = aerodynamic loading per unit area
L_{mnrs} = element of aerodynamic matrix, Eq. (17)
m,n,r,s = indices in the deflection series
M = Mach number, also maximum value of indices m,r
N = maximum value of indices n,s
N_x,N_{xy},N_y = midplane forces per unit length
p^* = reciprocal of $\bar{k}^{*2}$
q = dynamic pressure, $\frac{1}{2}\rho_a u^2$
Q,Q^* = dynamic pressure parameter, $2qa^3/\beta D\pi^4$ and $2qa^3 \cos^4\psi/\beta D\pi^4$, respectively
$\bar{R}_x^*,\bar{R}_{xy}^*,\bar{R}_y^*$ = nondimensional midplane force parameters, $N_xa^2 \cos^4\psi/D\pi^2$, $N_{xy}a^2 \cos^3\psi/D\pi^2$, $N_ya^2 \cos^2\psi/D\pi^2$, respectively
t = time
u = velocity of freestream
$w(x,y,t)$ = time-dependent deflection of panel
$W(x,y)$ = deflection surface of the panel
x,y,z = rectangular coordinate system, defined in Figs. 1 and 2
x_1,y_1 = oblique coordinates, defined in Figs. 1 and 2
$\bar{x},\bar{y}$ = rectangular coordinates, defined in Fig. 2
$X_r(\xi)$ = rth beam characteristic function in the ξ direction, see Eq. (10)
$Y_s(\eta)$ = sth beam characteristic function in the η direction, see Eq. (10)
β = $(M^2 - 1)^{1/2}$
ϵ_r,ϵ_s = beam eigenvalues
ρ = mass density of panel material
ρ_a = mass density of freestream air
ψ = angle of skew, defined in Figs. 1 and 2
λ,λ^* = dynamic pressure parameter, $2qa^3/\beta D$ and $2qa^3 \cos^4\psi/\beta D$, respectively
Λ = angle of yaw, defined in Fig. 2
ξ,η = nondimensional coordinates, x_1/a and y_1/b, respectively
ν = Poisson's ratio
ω = frequency of oscillation, rad/sec
δ_{mnrs} = Kronecker delta, = 1 for $m = r$ and $n = s$; = 0 for $m \neq r$ or $n \neq s$
∇^4 = biharmonic operator in rectangular coordinates, $\partial^4/\partial x^4 + 2\partial^4/\partial x^2\partial y^2 + \partial^4/\partial y^4$

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Introduction

ALTHOUGH the flutter problems of semi-infinite and finite rectangular panels, circular and elliptic panels which are initially in a flat, curved, or buckled condition with different support conditions and employing various aerodynamic theories, have been studied extensively,[1-3] the flutter problems of skew and trapezoidal panels that may be found in the construction of some of the swept back aerodynamic surfaces of high-speed vehicles have not received much attention. The actual boundary conditions due to the attachment of the skin to underlying support structure may be more nearly in the nature of elastic restraint. Although it is realistic to consider this boundary condition, the analytical treatment of the same is, admittedly, quite tedious and it is even more so for skew geometry. Consequently, it is usual to consider the ideal boundary conditions of simple support and clamping which would yield the two limiting values for the physical quantities of design interest. In Ref. 4, the flutter problem of simply supported skew panels under the action of in-plane forces is formulated. Approximate solution is obtained by using Lagrange's equations, and the results for unstressed panels have been presented therein. The results for stressed panels have been discussed in Ref. 5. With regard to clamped panels, the note by Kornecki,[6] based on a 4-mode analysis using Iguchi functions, appears to be the only published account so far, apart from the author's presentation[7] of the preliminary results. Recently, finite element methods also have been applied to panel flutter analyses.[8,9] Olson[8] considered the flutter problems of two-dimensional, simply supported and clamped panels, and obtained very good agreement with exact solutions by using 4–5 elements. In Ref. 9, the finite element method has been applied to panel flutter problems by deriving kinematically consistent aerodynamic influence coefficients using a parallelogram element. However, published results are not yet available for clamped skew panels. Also, no experimental results are available for this configuration so far.

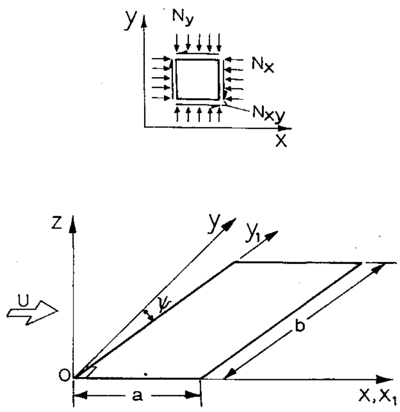

Fig. 1 Geometry of skew panel and the in-plane force system.

In this paper, the flutter problem of clamped skew panels subjected to in-plane forces is formulated. Approximate analysis is made by using the Galerkin method, expressing the flutter mode in terms of double series of beam characteristic functions in oblique coordinates. This study is complementary to our earlier investigation[4] on simply supported skew panels.

Theoretical Analysis

The skew panel is flat, clamped on all the four edges and is acted upon by in-plane forces. The geometry of the panel, the coordinate axes and the in-plane force system in terms of orthogonal components along with the assumed positive sign convention are all shown in Fig. 1. The panel is considered to be uniform, thin and isotropic. It is exposed to supersonic flow on the outer surface and to still air on the inner surface. Structural damping is ignored in the present analysis. The classical small deflection, thin plate theory is used. The governing differential equation can then be written as

$$D\nabla^4 w + N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} + \rho h w_{,tt} = l(x,y,t) \quad (1)$$

where the subscripts after a comma denote differentiation. In terms of oblique coordinates, the boundaries of the panel may be represented as

$$x_1 = 0, x_1 = a; \quad y_1 = 0, \quad y_1 = b \quad (2)$$

The two coordinate systems are related by the equations

$$x_1 = x - y \tan\psi; \quad y_1 = y \sec\psi \quad (3)$$

The two-dimensional static approximation is used for the lateral aerodynamic loading, as it has been shown to be reasonable for flat panels at sufficiently high Mach numbers.[10,11] The influence of yaw of the skew panel is also taken into account. Figure 2 shows a sketch of the skew panel in yaw along with the system of axes. The aerodynamic loading can then be written as

$$l(x,y,t) = -(2q/\beta)(\partial w/\partial \bar{x}) \quad (4)$$

The nondimensional coordinates

$$\xi = x_1/a \text{ and } \eta = y_1/b \quad (5)$$

are introduced for convenience.

For a plate which is clamped all around, the boundary conditions are

$$w = \partial w/\partial n = 0 \quad \text{on all the edges} \quad (6)$$

where n denotes the direction of the outward normal to the edge. The boundary conditions of the present problem are entirely of the "geometric" or "kinematic" type. It can be shown, by the use of Eqs. (3) and (5), that the boundary conditions, Eq. (6), reduce to the form

$$w = \partial w/\partial \xi = 0 \quad \text{on} \quad \xi = 0 \text{ and } 1 \quad (7a)$$

$$w = \partial w/\partial \eta = 0 \quad \text{on} \quad \eta = 0 \text{ and } 1 \quad (7b)$$

At the crtical flutter condition, with the motion simple harmonic, one can write

$$w(\xi,\eta,t) = R_e W(\xi,\eta)e^{i\omega t} \quad (8)$$

Substituting this into the equation obtained by using Eqs. (3–5) in Eq. (1) finally results in the equation

$$W_{,\xi\xi\xi\xi} + (a/b)^4 W_{,\eta\eta\eta\eta} + 2(1 + 2\sin^2\psi)(a/b)^2 W_{,\xi\xi\eta\eta} - 4\sin\psi(a/b)[W_{,\xi\xi\xi\eta} + (a/b)^2 W_{,\xi\eta\eta\eta}] + (a/b)^2 W_{,\xi\xi}(R_x^* - 2R_{xy}^*\sin\psi + R_y^*\sin^2\psi) + W_{,\eta\eta}R_y^*(a/b)^4 + 2W_{,\xi\eta}(a/b)^3(R_{xy}^* - R_y^*\sin\psi) - \bar{k}^{*2}W + \lambda^*[(\cos\Lambda - \sin\Lambda\tan\psi)W_{,\xi} + (a/b)\sin\Lambda\sec\psi W_{,\eta}] = 0 \quad (9)$$

The critical dynamic pressure λ^* at which flutter occurs is to be found from this equation as the lowest value of λ^* at which a nonzero $W(\xi,\eta)$ satisfying Eq. (9) and the boundary conditions, given by Eqs. (7), is possible.

An approximate solution of this problem is attempted by using the Galerkin method.[12] The deflection of the panel is expressed in terms of a double series of beam characteristic functions[16–18] in the form

$$W(\xi,\eta) = \sum_{r=1}^{M}\sum_{s=1}^{N} C_{rs}X_r(\xi)Y_s(\eta) \quad (10)$$

where $X_r(\xi)$, $Y_s(\eta)$ represent the normalized mode shapes of a uniform clamped-clamped beam.

Each term in the series, Eq. (10), fully satisfies the boundary conditions, Eqs. (7). These functions are tabulated by Young and Felgar,[13] and the integrals involving these functions and their derivatives are given by Felgar.[14] We define the integrals

$$I_{m,r}^{(1)} = \int_0^1 X_m X_r d\xi; \; I_{m,r}^{(2)} = \int_0^1 X_m X_r' d\xi; \; I_{m,r}^{(3)} = \int_0^1 X_m X_r'' d\xi; \; I_{m,r}^{(4)} = \int_0^1 X_m X_r''' d\xi \quad (11)$$

where primes denote differentiation with respect to the nondimensional variable ξ. Similar integrals involving $Y_n(\eta)$, $Y_s(\eta)$ are labeled as J integrals. The functions $X_m(\xi)$, $Y_n(\eta)$ are the same in the present problem since the boundary conditions on both the pairs of opposite edges are identical; hence, the J integrals have the same values as the corresponding I integrals. In Ref. 14 the formulas for these integrals have been given from which the numerical values are easily calculated. These values, along with the values for a few other combinations of commonly occurring boundary conditions are tabulated in Ref. 15.

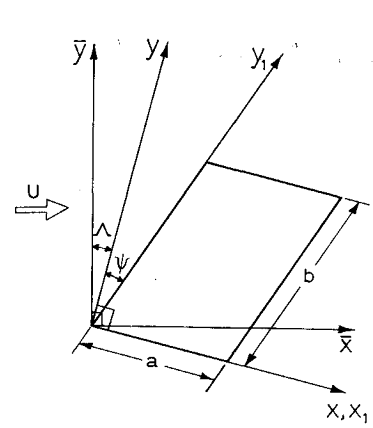

Fig. 2 Coordinate systems of panel in yaw.

Applying the Galerkin method, by substituting Eq. (10) in Eq. (9) and orthogonalizing the resulting error in the differential equation with respect to each of the product functions $X_m(\xi)Y_n(\eta)$ of Eq. (10), we get a set of homogeneous, linear, simultaneous, algebraic equations which can be written finally in the matrix notation as follows:

$$[E_{mnrs}]\{C_{rs}\} = \bar{k}^{*2}\{C_{rs}\} \tag{12}$$

where

$$E_{mnrs} = 1/\pi^4\{[\epsilon_r^4 + (a/b)^4\epsilon_s^4]\delta_{mnrs} + 2(1 + 2\sin^2\psi)(a/b)^2 I_{m,r}^{(3)}J_{n,s}^{(3)} - 4\sin\psi(a/b) \times [I_{mr}^{(4)}J_{ns}^{(2)} + (a/b)^2 I_{mr}^{(2)}J_{ns}^{(4)}]\} - \bar{R}_x^* H_{mnrs}^{(1)} - \bar{R}_{xy}^* H_{mnrs}^{(2)} - \bar{R}_y^* H_{mnrs}^{(3)} - Q^* L_{mnrs} \tag{13}$$

with

$$H_{mnrs}^{(1)} = -(1/\pi^2)I_{m,r}^{(3)}J_{n,s}^{(1)} \tag{14}$$

$$H_{mnrs}^{(2)} = 2/\pi^2[\sin\psi I_{m,r}^{(3)}J_{n,s}^{(1)} - (a/b)I_{m,r}^{(2)}J_{n,s}^{(2)}] \tag{15}$$

$$H_{mnrs}^{(3)} = -1/\pi^2[\sin^2\psi I_{m,r}^{(3)}J_{n,s}^{(1)} + (a/b)^2 I_{m,r}^{(1)}J_{n,s}^{(3)} - 2(a/b)\sin\psi I_{m,r}^{(2)}J_{n,s}^{(2)}] \tag{16}$$

and

$$L_{mnrs} = -[(\cos\Lambda - \sin\Lambda\tan\psi)I_{m,r}^{(2)}J_{n,s}^{(1)} + (a/b)\sin\Lambda\sec\psi I_{m,r}^{(1)}J_{n,s}^{(2)}] \tag{17}$$

Equation (12) represents the algebraic eigenvalue problem corresponding to the general problem of panel flutter of a clamped skew panel acted upon by in-plane forces N_x,N_{xy}, and N_y. The problems of free vibration, buckling under the action of N_x,N_{xy},N_y individually or in combination, and panel flutter of unstressed panels (i.e., with $\bar{R}_x^* = \bar{R}_{xy}^* = \bar{R}_y^* = 0$) are special cases of the preceding general equation by dropping the appropriate terms. The results of the free vibration and buckling calculations are reported in Refs. 16–18. In this paper, the panel flutter problem of unstressed panels is solved and the results thereof are presented.

The eigenvalues $\bar{k}^{*2}$ of the matrix $[E]$ represent the frequency parameter of vibration of the panel. With the static aerodynamic theory used, all eigenvalues of Eq. (12) are real for sufficiently small values of Q^*. For $Q^* = 0$, the problem posed by Eq. (12) is evidently a free vibration problem, and the resulting eigenvalues correspond to the in-vacuo frequencies of the panel. As Q^* is gradually increased, some of the eigenvalues approach each other and for a certain value of Q^*, two of them coalesce, forming an eigenvalue loop. If Q^* is increased further, these two would become complex, which means that the corresponding motion becomes a divergent oscillation. Thus, the value of Q^* at which two eigenvalues coalesce defines the critical value Q_{cr}^* for flutter. This criterion for critical flutter is well known and is commonly adopted in theoretical panel flutter analyses using static approximation.[10,11,19,20] Usually, it is the eigenvalues at the lower end of the spectrum which tend to coalesce earlier, leading to instability. The available library program was based on the iteration method.[21] Since iteration on Eq. (12) would first yield eigenvalues at the higher end of the spectrum, it is necessary to perform the iteration on the inverted matrix to get the eigenvalues at the lower end. The matrix equation is then given by

$$[E]^{-1}\{C\} = p^*\{C\} \tag{18}$$

where $p^* = 1/\bar{k}^{*2}$. The matrix iteration method becomes considerably slow when two eigenvalues are close to each other resulting, consequently, in an increase of computer time. Of course, this may be avoided by using other methods. However, as library routines for other methods were not readily available, the following modification was successfully effected to speed up the process. The iteration was performed after raising the original matrix to a convenient higher power thereby adequately effecting an artificial separation of the eigenvalues. After having found the eigenvalues of the raised matrix, the appropriate root of these is taken to yield the eigenvalues of the original matrix. The matrix equation may then be written as

$$[E]^{-\sigma}\{C\} = p^{\sigma*}\{C\} \tag{19}$$

where σ is the power to which the matrix is raised. A value of 6 was found to be quite appropriate. By this artifice, the time required to calculate the first four eigenvalues has been cut down to approximately half, in a few example cases, and without any significant loss of numerical accuracy of the eigenvalues. In subsequent calculations made on a CDC-3600 computer, a library routine based on the versatile QR-transformation method[21] was available, completely obviating the need for this artifice.

Table 1 Convergence study: clamped skew panel flutter analysis

a/b	ψ, deg	Λ, deg	M	N	Q_{cr}^*	Coalescence	Q_{cr}
1	0	0	2	2	6.35	1,2	6.35
			3	2	10.75	1,2	10.75
			3	3	10.45	1,2	10.45
			4	2	8.65	1,2	8.65
			4	3	8.60	1,2	8.60
			4	4	8.60	1,2	8.60
1	0	15	2	2	6.55	1,2	6.55
			3	2	10.15	1,2	10.15
			4	3	8.7	1,2	8.7
			4	4	8.75	1,2	8.75
1	15	0	2	2	6.05	1,2	6.95
			4	3	7.95	1,2	9.14
			4	4	7.92	1,2	9.11
1	30	0	4	4	6.35	1,2	11.3
			6	4	6.28	1,2	11.2
1	39.5	0	4	4	5.08	1,2	14.3
			6	4	4.91	1,2	13.9
1	45	0	4	3	3.95	1,2	15.8
			4	4	4.16	1,2	16.6
0.5	0	0	2	2	5.4	1,3	5.4
			2	3	5.4	1,4	5.4
			2	4	5.4	1,4	5.4
			3	2	7.52	1,3	7.52
			3	3	7.46	1,4	7.46
			3	4	7.46	1,4	7.46
			4	4	7.1	1,4	7.1
0.5	15	15	2	2	2.35	1,2	2.70
			4	4	3.19	1,2	3.67
0.5	30	0	4	4	3.03	1,2	5.39
			5	4	3.25	1,2	5.76
0.5	45	0	3	3	1.26	1,2	5.03
			4	4	1.41	1,2	5.64
			4	5	1.8	1,2	7.2
2	0	0	2	2	10.2	1,2	10.2
			4	3	15.4	1,2	15.4
			4	4	15.4	1,2	15.4

Details of Calculations

The results reported in this paper pertain to unstressed panels. Numerical calculations have been made for different combinations of side ratio a/b, angle of skew ψ, and angle of yaw Λ. A majority of the calculations have been made using 16-term series ($M = 4, N = 4$), resulting in a matrix of order 16 $\times$ 16. For some representative configurations of the panel, convergence was examined by increasing the number of terms. Originally, the computations were performed on the National Elliott 803 Digital Computer at the Bangalore Division of the Hindusthan Aeronautics Ltd. During the initial computations, the matrix has not been raised in some cases, but in the subsequent computations it was raised

Table 2 Results of clamped skew panel flutter analysis[a]

a/b	ψ, deg	Λ, deg	M	N	Q_{cr}	Coalescence	Q_{cr}	Remarks				
1	0	0	4	4	8.60	1,2	8.60	6.32[b]	8.97[c]	4.50[d]	9.1[e]	8.4[f]
	10	0	4	4	8.25	1,2	8.77	6.63				
	15	0	4	4	7.92	1,2	9.11	6.97				
	20	0	4	3	7.6	1,2	9.74	7.64				
	21.3	0	6	4	7.3	1,2	9.7	...				
	30	0	6	4	6.28	1,2	11.2	10.2				
	39.5	0	6	4	4.91	1,2	13.8					
	45	0	4	4	4.16	1,2	16.6					
	0	10	4	4	8.65	1,2	8.65					
	0	15	4	4	8.75	1,2	8.75					
	0	30	4	3	8.9	1,2	8.9					
	0	45	4	4	8.98	1,2	8.98					
	10	10	4	4	8.2	1,2	8.7					
	20	10	4	4	7.3	1,2	9.4					
	30	10	4	4	6.21	1,2	11.0					
	45	10	4	4	3.98	1,2	15.9					
	15	15	4	4	7.7	1,2	8.8					
	30	15	4	4	6.15	1,2	10.9					
	45	15	4	4	3.96	1,2	15.8					
	10	30	4	4	8.12	1,2	8.63					
	20	30	4	4	7.12	1,2	9.13					
0.5	0	0	4	4	7.1	1,4	7.1	5.23[b]	1.28[d]	7.5[e]	6.8[f]	7.2[g]
	15	0	4	4	4.36	1,2	5.01	4.08				
	20	0	4	4	3.78	1,2	4.85	4.08				
	30	0	5	4	3.25	1,2	5.76	4.46				
	45	0	4	5	1.8	1,2	7.2	9.04				
	0	10	4	4	5.21	1,2	5.21					
	0	15	4	4	4.65	1,2	4.65					
	0	30	4	4	3.35	1,2	3.35					
	0	45	4	4	2.62	1,2	2.62					
	15	15	4	4	3.19	1,2	3.67					
	30	15	4	5	2.4	1,2	4.27					
	45	15	4	5	1.68	1,2	6.72					
2	0	0	4	4	15.4	1,2	15.4	15.5[c]	14.1[d]	16.1[e]		
	15	0	4	4	15.6	1,2	18.0					

[a] All values except those from Ref. 27 are taken from graphs.
[b] Reference 6.
[c] Reference 22, but as quoted in Ref. 11.
[d] Reference 25.
[e] Reference 26.
[f] Reference 27.
[g] Reference 28.

to the sixth power, as described in the previous section. Subsequently, some computations were also made on the CDC-3600 computer at the Tata Institute of Fundamental Research, Bombay, with the program coded in FORTRAN. The results of some convergence studies are given in Table 1. In Table 2, the results for Q_{cr}^* for different configurations of the panel are given, along with the details regarding the number of terms in the series, the modes which are coalescing etc.

Discussion of Results

There are two considerations with regard to the applicability of the results obtained; one, from the point of view of the aerodynamic theory used, and the other, from the point of view of the convergence of the solution.

Considerations governing the applicability of the "static approximation" have been discussed in detail in Ref. 4 on the basis of the findings of Refs. 10, 11, 19, 22–24. The remarks made in Ref. 4 apply equally well to the present problem of clamped skew panels. They may be briefly summarized as follows: the results of the present paper with no midplane stresses would be applicable for Mach numbers of the order of 2 and when the ratio air mass/panel mass is small (which implies low values of aerodynamic damping). In any case, the actual limits of applicability of static approximation for skew panels can only be established by comparison with an analysis using at least the first-order aerodynamic theory,[24] if not by using the full, linearized, three-dimensional unsteady aerodynamics.[25]

The results of the convergence study are presented in Table 1. Convergence study has been made for a few representative configurations including, of course, configurations for which one may expect convergence problems. In general, the convergence is seen to be oscillatory in almost all the cases, as is typical in modal solutions in panel flutter problems.[10,20] This appears to be the case with the finite element method also.[8] For $a/b = 1$, $\psi = 30°$, $\Lambda = 0°$, and $a/b = 1$, $\psi = 39.5°$, $\Lambda = 0°$, it is seen that the $M = 4$, $N = 4$ solutions are about 1% and 3% higher, respectively, than $M = 6$, $N = 4$ solutions, indicating thereby that the convergence of the 16-term solution is satisfactory. For the cases $a/b = 1$, $\psi = 45°$, $\Lambda = 0°$ and $a/b = 0.5$, $\psi = 45°$, $\Lambda = 0°$, the convergence does not appear to be too satisfactory. These results, though not very complete, seem to indicate that, for $a/b = 1$, a 16-term solution is quite satisfactory up to 40°, and beyond 40°, it tends to become less satisfactory. For $a/b = 0.5$, on the other hand, convergence appears to gradually worsen with increasing skew angle. Thus, it appears that the 16-term result is adequate up to about 40° and more terms would be required for skew angles greater than 40° for better convergence. It is felt that a similar thing is likely to occur with regard to the angle of yaw. Hence, portions of the curves in Figs. 3–5 beyond 40° are shown by a dotted line.

From this table, it is also seen that for $\psi = 0°$, the increase of N for a fixed value of M has no influence on the Q_{cr}. This is as it should be, since the aerodynamic load depends only on the stream-wise slope, and any increase in the number of modes in the cross-stream direction is not helpful in representing the stream-wise deflection shape more accurately. In fact, it may be remarked here that for $\psi = 0°$ (i.e., rectangular plates), and $\Lambda = 0°$, $N = 1$ is quite adequate in itself and one need not have used a larger N at all.

The results for several configurations of the panel are all finally presented in Table 2 along with other results, where available, for comparison. For a square plate, results of earlier investigations of Houbolt,[22] Cunningham,[25] and Ketter[26] (isotropic plate as a particular case of orthotropic plate) are also reported. Our result agrees very well with Houbolt's value obtained by a suitable interpretation of the exact solution for two-dimensional panels. It is gratifying to note, in particular, that agreement between our calculation, which ignored aerodynamic damping, and Houbolt's calculation, which included aerodynamic damping, is good. This suggests that, for this configuration, the use of static approximation is quite justified. In fact, Ref. 22 has also clearly shown the very slow variation of critical dynamic pressure with aerodynamic damping in the high Mach number range. The agreement with Ketter's value is also quite satisfactory; in fact, the present analysis using beam characteristic functions for an isotropic skew plate in the limit of skew angle tending to zero becomes identical with Ketter's analysis for clamped orthotropic rectangular plate using beam characteristic functions in the limit of orthotropy tending to isotropy and with aerodynamic damping ignored. The difference is probably due to the different number of terms used in the two analyses and the aerodynamic damping, apart from the fact that Ketter's values are reduced from a graph. Also, the result from Ref. 27 using polynomial approximation for semi-infinite panels, and deriving therefrom, the results for finite panels on the lines of Houbolt's analysis[22] compare well. Kornecki's 4-term result is too far on the lower side and Cunningham's[25] is even farther. Remarks about this comparison as well as others with regard to Cunningham's results are made later in this section. For other skew angles, only Kornecki's 4-term results are available which become increasingly approximate as the skew angle increases.

The results for $\psi = 21.3°$ and $\psi = 39.5°$ are also reported. The flutter calculation at these skew angles is made to ensure that no spurious dips in the critical dynamic pressure values result (as in Ref. 19) due to the frequency crossings observed between different modes in the free vibration characteristics of these panels.[16,17] It should be noted that the coalescing modes are only 1 and 2 in these two cases also.

The results for $a/b = 0.5$ are also presented in the same table along with available results for comparison. For $a/b = 0.5$, $\psi = 0°$, $\Lambda = 0°$, coalescence has been seen to be between the first and the fourth eigenvalues, as it should be, similar to the simply supported case. The comparison with Refs. 26 and 27 is quite fair. The value from Ref. 28, for $a/b = 0.46$, is also given and the comparison is quite fair

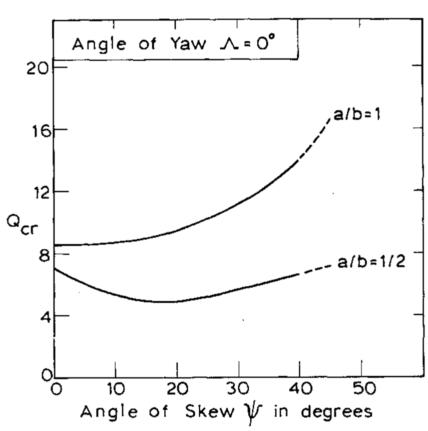

Fig. 3 Critical dynamic pressure vs angle of skew ψ ($\Lambda = 0°$).

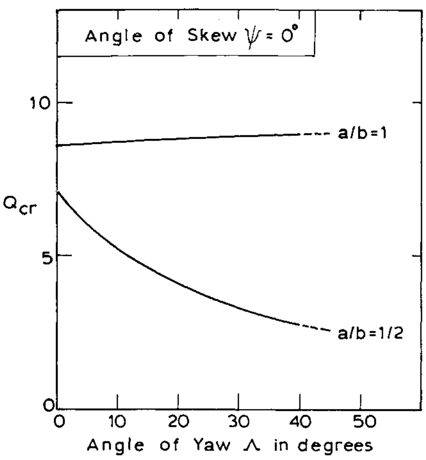

Fig. 4 Critical dynamic pressure vs angle of yaw Λ ($\psi = 0°$).

indeed. Houbolt's exact solution for semi-infinite panel is also given; the closeness of Q_{cr} for $a/b = 1/2$ with the result for semi-infinite panel is noteworthy. Kornecki's[6] value is again quite low and Cunningham's[25] is even lower still. For other skew angles, only Kornecki's results are available for comparison, and they continue to be on the lower side except for $\psi = 45°$.

In the case of $a/b = 2$, calculations for the full range of ψ have not been made. The results obtained for $\psi = 0°$ and 15° only are presented. The comparison with other available results is quite fair. It may be particularly noticed that Cunningham's[25] result in this case is close to our value as well as the values of Houbolt[22,11] and Ketter,[26] obtained by using two-dimensional quasi-steady theory. This is in contrast with the comparison with regard to $a/b = 1$ and 1/2. This is to be expected since Cunningham's result quoted for $a/b = 2$ is for $M = 2$, a value of Mach number for which the static approximation is known to be quite good, whereas in the case of $a/b = 1$ and 1/2, M is equal to 1.3, a value of Mach number in a range where static approximation is inapplicable. Thus, these comparisons confirm the known result that the three-dimensional and unsteady effects will have to be considered in the low Mach number range for $a/b \leq 1$ (see, for example, discussion on Fig. 1 of Ref. 11).

In comparison with the results for simply supported panels,[4] the critical dynamic pressure values for clamped panels are higher, as they ought to be, by virtue of the fact that a panel that is clamped on all edges is much stiffer than the simply supported one.

The results are finally plotted in Figs. 3–5. In Fig. 3, Q_{cr} vs ψ is plotted for $a/b = 1$ and 1/2 for $\Lambda = 0°$. For

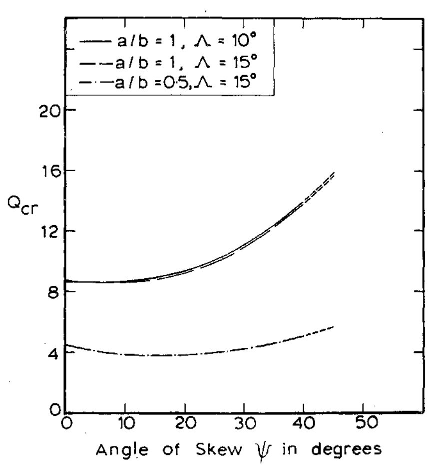

Fig. 5 Critical dynamic pressure vs angle of skew ψ.

$a/b = 1$, Q_{cr} increases monotonically, whereas for $a/b = 1/2$, it decreases up to about 20° before beginning to increase. In Fig. 4, Q_{cr} vs Λ is plotted for $a/b = 1$ and $1/2$ for $\psi = 0°$ (i.e., rectangular plates). For $a/b = 1$, Q_{cr} increases slightly, whereas for $a/b = 1/2$ it decreases with Λ. In other words, for square plates, angle of yaw is mildly stabilizing, whereas for $a/b = 1/2$, it is destabilizing. This is in agreement with the results for simply supported rectangular plates (see Ref. 19). In Fig. 5, Q_{cr} vs ψ is plotted for $a/b = 1$, $\Lambda = 10°$; $a/b = 1$, $\Lambda = 15°$; and $a/b = 0.5$ and $\Lambda = 15°$. The difference between the former two cases is indeed imperceptible. For $\psi = 0°$, the Q_{cr} for $\Lambda = 15°$ is slightly higher, but for all other ψ's it is slightly on the lower side. Q_{cr} is seen to decrease very slightly up to about a $\psi = 12°$ before beginning to increase. Also, for $a/b = 0.5$, $\Lambda = 15°$, Q_{cr} decreases initially up to about 30° and increases thereafter. From Figs. 3 and 5, it is also seen that for skew panels, angle of yaw is destabilizing for both $a/b = 1$ and $1/2$, which is in agreement with the results for the simply supported case (see Ref. 4).

Conclusions

The results for unstressed clamped skew panels, obtained by the Galerkin method employing a double series of beam characteristic functions and using the two-dimensional static approximation for the aerodynamic loading, are presented. As two-dimensional static approximation for the aerodynamic loading is used, the results of the present paper are applicable at Mach numbers of the order of 2 and when the ratio air-mass/panel mass is small, which implies low aerodynamic damping. A comparison with some of the available results in a few cases confirms the known result that the three-dimensional and unsteady effects will have to be considered in the low Mach number range for $a/b \leq 1$. A sixteen-term solution appeared to be quite reasonable up to about 40° in the case of $a/b = 1$. For higher skew angles as well as other side ratios, more terms are required to establish better convergence. For $a/b = 1$, it is found that the critical dynamic pressure parameter Q_{cr} increases monotonically with the angle of skew ψ, whereas for $a/b = 1/2$ it decreases initially up to a skew angle of about 20° before beginning to increase. Angle of yaw is found to be mildly stabilizing for square plates and quite destabilizing for $a/b = 1/2$, which is in agreement with the results for simply supported rectangular plates. For skew panels, angle of yaw is found to be destabilizing for both $a/b = 1$ and $1/2$, which is in agreement with the results for the simply supported case. The critical dynamic pressure values for clamped skew panels are higher than the values for simply supported panels, as may be expected.

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